MATH PROBLEM SOLVING
FOR MIDDLE SCHOOL STUDENTS WITH DISABILITIES

ABOUT THE AUTHOR

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Marci bought a school shirt for $4.95 and gym shorts for $5.59. How much change should she get back if she paid with a $20 bill?

An airline agent checked a bag that weighed 35 pounds, another that weighed 4 pounds less than the first bag, and a third that weighed 13 pounds less than the second. How many pounds were checked?

A plane flew 1,485 miles in three hours. What was the average distance flown each hour?

These are typical textbook problems that middle school students should be able to solve with ease. However, many students, especially students with learning disabilities (LD), have difficulty solving even simple mathematical word problems like the ones above. These students most likely have not acquired the skills and strategies needed to "decide what to do" when they are confronted with problems in their math textbooks or cannot apply the skills and strategies they do have to solve math problems in school and in their daily lives. This brief focuses on teaching middle school students how to solve mathematical word problems. The following frequently asked questions provide the framework for the brief.

What is mathematical problem solving?

How do good problem solvers solve math problems?

Why is it so difficult to teach students to be good math problem solvers?

What is the content of math problem solving instruction?

What are effective instructional procedures for teaching math problem solving?

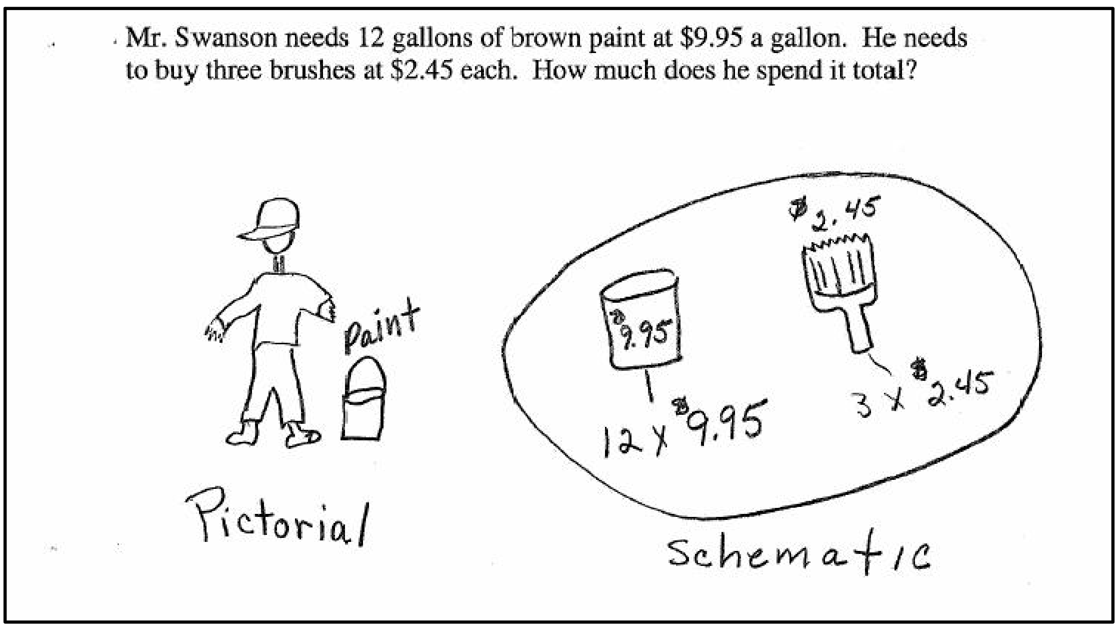
Solve It!, a program validated for use with students with LD, is then described, and a sample lesson is provided. Finally, an example of how to modify *Solve It!* for students with other types of disabilities, such as spina bifida and Asperger's Syndrome, is provided.

WHAT IS MATHEMATICAL PROBLEM SOLVING?

Mathematical problem solving is a complex cognitive activity involving a number of processes and strategies. Problem solving has two stages: problem representation and problem execution. Successful problem solving is not possible without first representing the problem appropriately. Appropriate problem representation is the basis for understanding the problem and making a plan to solve the problem. Students who have difficulty representing math problems will have difficulty solving them. These students either have not acquired problem representation strategies or do not know how to use them appropriately.

To illustrate, visualization is a very powerful representation strategy. Many students do not develop the ability to use visual representation automatically during math problem solving. These students need explicit instruction in how to use visualization to represent problems. Other students may use visualization, but apply it inappropriately, and, thus, ineffectively. Teaching mathematical problem solving is a challenge for teachers, who generally rely on mathematics textbooks to guide instruction. Most mathematics textbooks simply instruct students to draw a picture or make a diagram using the information in the problem. When doing so, however, students who have difficulty solving math word problems usually draw a picture of the problem without considering the relationships among the problem components as a result, they still do not understand the problem, and therefore cannot make a plan to solve it. So, it is not simply a matter of “drawing a picture or making a diagram;” rather, it is the type of picture or diagram that is important. Effective visual representations, whether on paper or in one’s imagination, show the relationships among the problem parts. These are called schematic representations (van Garderen & Montague, 2003). Poor problem solvers tend to make immature representations that are more pictorial than schematic in nature. The illustration below shows the difference between a pictorial and a schematic representation of a mathematical problem.

Figure 1: Pictorial and Schematic Representations



Other cognitive processes needed for successful mathematical problem solving include reading the problem for understanding, paraphrasing the problem by putting it into your own words, hypothesizing or making a plan to solve the problem, estimating or predicting the outcome, computing or doing the arithmetic, and checking to make sure the plan was appropriate and the answer is correct (Montague, 2003; Montague, Warger, & Morgan, 2000). Mathematical problem solving requires not only cognitive processes, like visualization and estimation, but also requires self-regulation strategies. As they solve problems, students must tell themselves what to do; ask questions; and evaluate, monitor, and verify what they do.

WHAT DO GOOD PROBLEM SOLVERS DO?

Good problem solvers use a variety of processes and strategies as they read and represent the problem before they make a plan to solve it. First, they **READ the problem for understanding.** As they read, they use comprehension strategies to translate the linguistic and numerical information in the problem into mathematical notations. For example, good problem solvers may read the problem more than once and may reread parts of the problem as they progress and think through the problem. They use self-regulation strategies by asking themselves if they understood the problem. They **PARAPHRASE the problem by putting it into their own words.** They identify the important information and may even underline parts of the problem. Good problem solvers ask themselves what the question is and what they are looking for.

VISUALIZING or drawing a picture or diagram means developing a schematic representation of the problem so that the picture or image reflects the relationships among all the important problem parts. Using both verbal translation and visual representation, good problem solvers not only are guided toward understanding the problem, but are also guided toward developing a plan to solve the problem. This is the point at which students decide what to do to solve the problem. They have represented the problem and they are now ready to develop a solution path. They **HYPOTHESIZE** by thinking about logical solutions and the types of operations and number of steps needed to solve the problem. They may write the operation symbols as they decide on the most appropriate solution path and the algorithms they need to carry out the plan. They ask themselves if the plan makes sense given the information they have. Good problem solvers usually **ESTIMATE or predict the answer** using mental calculations or may even quickly use paper and pencil as they round the numbers up and down to get a "ballpark" idea. They are now ready to **COMPUTE.** So they tell themselves to do the arithmetic and then compare their answer with their estimate. They also ask themselves if the answer makes sense and if they have used all the necessary symbols and labels such as dollar signs and decimals. Finally, they **CHECK** to make sure they used the correct procedures and that their answer is correct.

WHY IS IT SO DIFFICULT TO TEACH STUDENTS TO SOLVE MATH PROBLEMS?

Students who are poor mathematical problem solvers, as most students with LD are, do not process problem information effectively or efficiently. They lack or do not apply the resources needed to complete this complex cognitive activity. Generally, these students also lack metacognitive or self-regulation strategies that help successful students understand, analyze, solve, and evaluate problems. To help these students become good problem solvers, teachers must understand and teach the cognitive processes and metacognitive strategies that good problem solvers use. This is the **CONTENT** of math problem solving instruction. Teachers must also use instructional **PROCEDURES** that are research-based and have proven effectiveness. These procedures are the basis of **COGNITIVE STRATEGY INSTRUCTION**, which has been demonstrated to be one of the most powerful interventions for students with LD (Swanson, 1999).

WHAT IS THE CONTENT OF MATH PROBLEM SOLVING INSTRUCTION?

The previous sections described the content of math problem solving instruction as the cognitive processes and metacognitive strategies that good problem solvers use to solve mathematical problems. Students learn how to use these processes and strategies not only effectively, but efficiently as well. The following chart lists the processes and their accompanying self-regulation strategies that facilitate application of the processes (Montague, 2003).

MATH PROBLEM SOLVING PROCESSES AND STRATEGIES

READ (for understanding)
Say: Read the problem. If I don't understand, read it again.
Ask: Have I read and understood the problem?
Check: For understanding as I solve the problem.

PARAPHRASE (your own words)
Say: Underline the important information. Put the problem in my own words.
Ask: Have I underlined the important information? What is the question? What am I looking for?
Check: That the information goes with the question.

VISUALIZE (a picture or a diagram)
Say: Make a drawing or a diagram. Show the relationships among the problem parts.
Ask: Does the picture fit the problem? Did I show the relationships?
Check: The picture against the problem information.

HYPOTHESIZE (a plan to solve the problem)
Say: Decide how many steps and operations are needed. Write the operation symbols (+, -, x, and /).
Ask: If I …, what will I get? If I …, then what do I need to do next? How many steps are needed?
Check: That the plan makes sense.

ESTIMATE (predict the answer)
Say: Round the numbers, do the problem in my head, and write the estimate.
Ask: Did I round up and down? Did I write the estimate?
Check: That I used the important information.

COMPUTE (do the arithmetic)
Say: Do the operations in the right order.
Ask: How does my answer compare with my estimate? Does my answer make sense? Are the decimals or money signs in the right places?
Check: That all the operations were done in the right order.

CHECK (make sure everything is right)
Say: Check the plan to make sure it is right. Check the computation.
Ask: Have I checked every step? Have I checked the computation? Is my answer right?
Check: That everything is right. If not, go back. Ask for help if I need it.

WHAT ARE EFFECTIVE INSTRUCTIONAL PROCEDURES FOR TEACHING MATH PROBLEM SOLVING?

Explicit Instruction

Explicit Instruction, the basis of cognitive strategy instruction, incorporates research-based practices and instructional procedures such as cueing, modeling, verbal rehearsal, and feedback. The lessons are highly organized and structured. Appropriate cues and prompts are built in as students learn and practice the cognitive and metacognitive processes and strategies. Each student is provided with immediate, corrective, and positive feedback on performance. Overlearning, mastery, and automaticity are the goals of instruction. Explicit instruction allows students to be active participants as they learn and practice math problem solving processes and strategies. This approach emphasizes interaction among students and teachers.

Cognitive strategy instruction uses a guided discussion technique to promote active teaching and learning. Students are engaged from the very beginning through an initial discussion of the importance of mathematical problem solving and their individual performance on a baseline test. With the teacher, they set individual performance goals and make a commitment to becoming a better problem solver. The instructional procedures that are the basis of cognitive strategy instruction are described next.

Verbal Rehearsal

Before students practice using the cognitive processes and self-regulation strategies, they must first memorize them by using verbal rehearsal. This is a memory strategy that enables students to recall automatically the math problem solving processes and strategies. Frequently, acronyms are created to help students remember as they verbally rehearse and internalize the labels and definitions for the processes and strategies. For math problem solving, the acronym **RPV-HECC** was created (**R** = Read for understanding, **P** = Paraphrase in your own words, **V** = Visualize – draw a picture or diagram, **H** = Hypothesize – make a plan, **E** = Estimate – predict the answer, **C** = Compute – do the arithmetic, **C** = Check – make sure everything is right). Cues and prompts are used to help students as they memorize the processes and their definitions. The goal is for students to recite from memory all processes and name the self-regulation strategies (SAY, ASK, CHECK). When students have memorized the processes for math problem solving, they can cue other students and the teacher as they begin to use the processes and strategies to solve problems.

Process Modeling

Process modeling is thinking aloud while demonstrating an activity. For mathematical problem solving, this means that the problem solver says everything he or she is thinking and doing while solving a problem. When students are first learning how to apply the processes and strategies, the teacher demonstrates and models what good problem solvers do as they solve problems. Students have the opportunity to observe and hear how to solve mathematical problems. Both correct and incorrect problem solving behaviors are modeled. Modeling of correct behaviors helps students understand how good problem solvers use the processes and strategies appropriately. Modeling of incorrect behaviors allows students to learn how to use self-regulation strategies to monitor their performance and locate and correct errors. Self-regulation strategies are learned and practiced in the actual context of problem solving. When students learn the modeling routine, they then can exchange places with the teacher and become models for their peers. Initially, students will need plenty of prompting and reinforcement as they become more comfortable with the problem solving routine. However, they soon become proficient and independent in demonstrating how good problem solvers solve math problems. One of the instructional goals is to gradually move students from overt to covert verbalization. As students become more effective problem solvers, they will begin to verbalize covertly and then internally. In this way, they not only become more effective problem solvers, but they also become more efficient problem solvers.

Visualization

Visualization is critical to problem representation. It allows students to construct an image of the problem on paper or mentally. Students must be shown how to select the important information in the problem and develop a schematic representation. To do this, teachers model how to draw a picture or make a diagram that shows the relationships among the problem parts using both the linguistic and numerical information in the problem. These visual representations can take many forms and will vary from student to student. Students may use a variety of visual representations such as pictures, tables, graphs, or other graphic displays. Initially, students must be told to use paper and pencil and later, as they become more proficient, they will progress to mental images. Interestingly, if the problem is novel or challenging, they frequently return to conscious application of processes and strategies, which is typical of good problem solvers.

Role Reversal

Role reversal is an important instructional activity that promotes independent learning. As students become familiar with the math problem solving routine, they can take on the role of teacher as model and actually change places with the teacher. They may use an overhead projector just as the teacher did and engage in process modeling to demonstrate that they can effectively apply the cognitive and metacognitive processes and strategies they have learned. Other students can prompt or ask questions for clarification. In this way, students learn to think about, explain, and justify their visual representations and their solution paths. Teachers may also take the role of the student who then guides the "student as teacher" through the process. This interaction allows students to appreciate that there is usually more than one correct solution path for a math problem; that is, problems can be solved in a variety of ways.

Peer Coaching

Peer partners, teams, and small problem solving groups give students opportunities to see the different ways in which their classmates approach mathematical problems, use cognitive and metacognitive processes and strategies and represent and solve problems. Students gain a broader perspective on the problem solving process and begin to realize that there is more than one way to solve a problem. Students become more flexible and open-minded thinkers as a result. With their partners or groups, students are encouraged to discuss the problems and work toward common solutions while appreciating the differences in approaches to each problem. This is also an opportunity to continue explaining and clarifying their choices. When students reach their performance goals and demonstrate mastery, novel or "real life" problems like the following can be introduced (Montague, 2003).

Novel Mathematical Problem for Partner, Team, or Group Problem Solving

Your dog is a yellow Labrador, and his name is Sylvester. He likes to be outside during the cool winter months, but he needs a doghouse that is comfortable and roomy. Design a doghouse and figure the cost of the materials needed to build it.

Performance Feedback

Performance feedback is critical to the success of the program. Progress checks are given throughout the program to determine mastery of the cognitive and metacognitive processes and strategies and performance on math problem solving tests. Students graph their progress to visually display their performance. Teachers carefully analyze performance during practice sessions and on mastery checks and provide each student with immediate, corrective feedback. Appropriate use of processes and strategies is reinforced continuously until students become proficient. Students need to know the specific behaviors for which they are praised so they can repeat these behaviors. Praise should be honest. Students should be taught how to give and receive reinforcement and should be given plenty of opportunities to practice doing it. The goal is to teach students to monitor, evaluate, and reinforce their problem solving approaches.

Distributed Practice

Distributed practice is the cornerstone for ensuring that students maintain the skills they have learned. To become good math problem solvers, students learn to use the processes and strategies that successful problem solvers use. As a result, their math problem solving skills and performance levels improve. However, to achieve high performance, students must be given ample opportunity to practice when they first learn the math problem solving routine and then, to maintain high performance, they must continue to practice intermittently over time. They may practice individually or in teams or small groups. They should be involved in solving a range of problems from textbook type problems to problems encountered in real life. Discussion about strategies, error monitoring, and alternative solutions is essential.

Mastery Learning

Prior to instruction, a pretest is given to determine baseline performance levels of individual students. During instruction, periodic mastery checks are given to monitor student progress over time and to determine effectiveness of the program. If students are not making sufficient progress, modifications to the program to ensure success must be made. Following instruction, periodic maintenance checks are provided. If students do not meet criterion on maintenance checks, booster sessions must be provided to improve performance levels to mastery. Booster sessions are brief lessons to review and refresh what students have previously learned and mastered.

Solve It! A Validated Math Problem Solving Program

Solve It! (Montague, 2003) is a curriculum designed to help middle and secondary school students who have difficulty solving mathematical problems. *Solve It!* teaches students the necessary cognitive and metacognitive processes and strategies that good problem solvers use. The processes and strategies were identified through a review of literature and a process-task analysis of problem solving. They were later validated as an effective problem solving cognitive routine in a series of studies with middle and secondary students with learning disabilities (Montague, 1992; Montague, Applegate, & Marquard, 1993; Montague & Bos, 1986). These studies demonstrated the effectiveness of the program. Following instruction, the students with learning disabilities performed as well as their average achieving peers. Generally, students maintained strategy use and improved performance for several weeks following instruction. When performance declined for some students, brief booster sessions consisting of review and practice were provided to help them return to mastery level. The research-based program was designed for easy inclusion in a standard mathematics curriculum. Students who are poor math problem solvers experience success at the outset and rapidly improve in problem solving performance. Students also develop a more positive attitude toward problem solving, an interest in mathematics and problem solving, independence as learners, and confidence in their ability to solve math problems.

Solve It! uses sequenced scripted lessons to ensure that the content is covered and research-based instructional procedures are implemented. Students are explicitly taught how to apply the cognitive processes and self-regulation strategies in the context of math problem solving. Prior to implementation, students are given pretests to determine their baseline performance level. Additionally, an informal assessment tool, the *Math Problem Solving Assessment-Short Form (MPSA-SF)*, is included to analyze students' knowledge and use of problem solving processes and strategies (Montague, 1996).

Solve It! A Sample Lesson

Solve It! lessons have instructional goals and behavioral objectives that reflect the content of the lesson. Materials are listed that indicate the instructional charts, practice problems, activities, and cue cards needed. Explicit instructional cues help the teacher pace the lesson by indicating which procedures to use and when to use them. The lesson script is divided into several steps. During Lessons 1-3, students learn the problem-solving routine (see page 7). Practice sessions ensure that students' performance improves to the criterion (e.g., at least 70% correct on math problem solving mastery checks). Reinforcement and review are emphasized to help students maintain strategy use and improved performance over time. Lessons are summarized below.

Lesson 1

The teacher guides a discussion with students about mathematical problem solving and why it is important to be a good problem solver.

Solve It! is described for students and the master class chart is presented.

Students practice verbalizing the processes and strategies by reading through the charts individually and as a group using choral reading techniques.

The teacher demonstrates how to use the comprehensive strategy to solve math word problems using process modeling.

Students are given study booklets.

Lesson 2

Students are tested for mastery of the seven cognitive processes. They recite from memory the names and descriptions of the processes.

The group practices recitation.

Individual students then take turns reciting the processes from memory.

Students are cued using the acronym (RPV-HECC) and the Master Class Charts posted on the walls of the classroom.

The teacher again demonstrates problem solving using process modeling.

Lessons 3 through 5

Students are tested for mastery of the processes (100% criterion).

The group recites all processes and the SAY, ASK, CHECK strategies.

Students solve a practice problem individually at their seats. They are told to think aloud and verbalize the processes and strategies as they solve the problem.

The teacher or a student models the correct solution.

Students and the teacher assist the problem solver by verbalizing the processes and strategies as they work through the problem.

The teacher leads the group in rehearsal activities.

The criteria for moving to Lesson 6 are that all students in the group meet the mastery criterion (100%) for recitation of the cognitive processes from memory, that all students understand and are able to use the SAY, ASK, CHECK strategies, and that all students are able to work through practice problems with relative comfort and confidence. Students who do not meet criteria repeat lessons 3 through 5.

Lesson 6

Students complete the first practice set of ten math problems one by one. They are cued to use the strategy, to use the Master Class Charts or their study booklets, and to think aloud.

After finishing each problem, either the teacher or a student models the correct solution.

Lesson 7

Students solve all 10 problems on the practice set.

The teacher and students model correct solutions for the problems.

Questions and discussion are encouraged.

Lesson 8

The first Progress Check (test of 10 problems) is administered.

Students model solutions for their classmates.

Students grade their papers.

Scores are plotted by students on their performance graph.

Practice sessions and progress checks are alternated until students reach criterion for mastery. The following vignette illustrates how *Solve It!* is implemented in a general education math class.

Mr. Wright's Math Class

Mr. Wright has 22 students in his seventh grade remedial math class. Six students have identified learning disabilities and receive resource room support. All of the students have difficulty solving mathematical word problems. Mr. Wright has been using *Solve It!* with these students. During Lessons 1 through 3, students were introduced to the processes and strategies, and they observed Mr. Wright as he solved math problems. By Lesson 4, all students reached 100% criterion in recitation of the cognitive processes from memory. They also were comfortable with the SAY, ASK, CHECK procedures and were less reliant on the wall charts and their study booklets. Mr. Wright had modeled problem solving for the students several times during the previous lessons. On occasion, individual students "guided" him through the process. Mr. Wright is beginning Lesson 4. He plans to model a solution one more time before students solve problems on their own.

He places a transparency of the math problem on the projector.

Mr. Wright: Watch me say everything I am thinking and doing as I solve this problem.

Mr. Swanson needs 12 gallons of brown paint at $9.95 a gallon. He needs to buy three brushes at $2.45 each. How much does he spend in total?

First, I am going to **read** the problem for understanding.

SAY: Read the problem. Okay, I will do that. (Mr. Wright reads the problem.) If I don't understand it, I will read it again. Hm, I think I need to read it again. (He reads the problem again.)

ASK: Have I read and understood the problem? I think so.

CHECK: For understanding as I solve the problem. Okay, I understand it.

Next, I am going to **paraphrase** by putting the problem into my own words.

SAY: Put the problem into my own words. This guy is buying 12 cans of paint and three brushes. Paint is $9.95 and brushes are $2.45 each. How much altogether? Underline the important information. I will underline <u>12 gallons</u> and <u>$9.95 a gallon</u> and <u>three brushes</u> and <u>$2.45 each</u>.

ASK: Have I underlined the important information? Let's see, yes I did. What is the question? The question is "how much did he spend in total?" What am I looking for? I am looking for the total amount of money for the paint and brushes.

CHECK: That the information goes with the question. I have the number of gallons he needs and the number of brushes and the cost for each. I need to find the total amount he spent for everything.

Then I will **visualize** by making a drawing or a diagram.

SAY: Make a drawing or a diagram. Hm, I will draw a bucket and label it 12 x $9.95 and draw a brush and label it $2.45 x 3 and draw a big circle around them to tell myself that it is the total. (See Figure 1).

ASK: Does the picture fit the problem? Yes, I believe it does tell the story.

CHECK: The picture against the problem information. Let me make sure I wrote the correct numbers: 12 x $9.95 and 3 x $2.45. Yes, I did.

Now I will **hypothesize** by making a plan to solve the problem.

SAY: Decide how many steps and operations are needed. Let me see. First I need to get the total amount for the paint and the total amount for the brushes. Then I need to add the two amounts together. Okay, $9.95 x 3, whoops, x 12. Okay, $9.95 x 12 and $2.45 x 3. So, multiply, multiply, and then add. Now I will write the operation symbols: X, X, +.

ASK: If I multiply $9.95 by 12, I will get the total amount for the paint, and then I will multiply $2.45 by 3 and get the total for the brushes. Then I will add both amounts to get the total amount he paid for the paint and brushes. How many steps are needed? 3 steps.

CHECK: That the plan makes sense. If not, ask for help. It makes sense.

Next I need to **estimate** by predicting the answer.

SAY: Round the numbers, do the problem in my head, and write the estimate. Round $9.95 to $10 and 12 cans to 10. 10 X $10 is $100. Round $2.45 to $3 X 3 brushes = $9. $100 plus $9 is $109, my estimated answer. My answer should be around $109.

ASK: Did I round up and down? Yes, I did. Did I write the estimate? Yes.

CHECK: That I used all the important information. Three steps. Okay.

Now I **compute** by doing the arithmetic.

SAY: Do the operations in the right order. Okay, first multiply $9.95 X 12. Okay (does the arithmetic thinking aloud), $119.40. Then multiply (does the arithmetic thinking aloud) $2.45 X 3 = $7.35. Then add $119.40 and $7.35 (does the arithmetic thinking aloud) for a total of $126.75, my answer.

ASK: How does my answer compare with my estimate? Hm, not bad. I rounded mostly down so my estimate would be less. Does my answer make sense? Yes, he paid $126.75 for the paint and brushes. Are the decimals or money signs in the right places? Yes, in place.

CHECK: That all the operations were done in the right order. X, X, +. Yes, they were.

Okay, now I really get to **check** to see if the answer is correct.

SAY: Check the computation. Let's see. I will reverse the order to multiply and then check the addition or I could divide to check the computation (demonstrate checking the computation).

ASK: Have I checked every step? Yes. Have I used the right numbers (returns to the problem and checks the numbers again). Yes, I have used the right numbers. Have I checked the computation? Yes, it's right. Is my answer right? Yes, the answer is right. CHECK: Now I will check myself again. I did everything correctly. The answer is right. I do not need to go back to the problem, and I do not need help.
Students then are given a problem to solve and are told to use the processes and strategies and to think out loud just as the teacher did. A student is then selected to model the solution with assistance from

Mr. Wright. Although Mr. Wright knows the program is effective for his students, he has several concerns.

Mr. Wright's Concerns

Assessing students individually with the MPSA-SF may not be feasible with large groups of students. If instructional aides are not available, the teacher may not be able to give the MPSA-SF to all students.
Individualizing instruction may be difficult given the large numbers of students enrolled in most middle school teachers' math classes. Class size can range from 25 to 40 students and teachers usually teach at least five classes. Enlisting the aid of the resource teacher to assist with instruction may be necessary.
Students may need to be grouped for instruction because some students may be good problem solvers already and may not need *Solve It!* instruction. Identifying the students who need instruction and then grouping for instruction based on the various levels in the class can be a challenge for a math teacher.

General education math teachers often feel unprepared to teach students who are in special programs. They may not feel confident that students can learn how to think differently and become good problem solvers.

Finding time to talk with the resource teacher for students in special education can be difficult. Also, teachers often do not coordinate resource room instruction with the general education math curriculum. Communication between teachers sometimes can be difficult.

Teachers may need to develop the knowledge and skills required to implement a program like *Solve It!* successfully. Because the program is intense and highly interactive, teachers may need professional development to learn the instructional procedures that are the foundation of cognitive strategy instruction.

Teachers may not be familiar with the research that supports cognitive strategy instruction and its components as well as the instructional procedures.

Modifying *Solve It!* for Students with Other Types of Disabilities

Students with other types of disabilities frequently display cognitive characteristics that resemble those of students with LD. However, their cognitive deficits may be more or less severe or may vary in some unique way from those of students with LD. In many cases, though, there seem to be more similarities than differences. For example, students with spina bifida have long-term memory, visual-spatial, and self-regulation problems that adversely affect their ability to comprehend text and do mathematics (Mesler, 2004). Children with chronic illnesses such as children surviving cancer, who have undergone intrusive medical treatments, often display delayed but prolonged attention, short-term memory, and other cognitive problems that interfere with school success (Bessell, 2001). Students with traumatic brain injury and Asperger's Syndrome also have cognitive deficits that are similar to students with LD. Because of these similarities, it seems reasonable to assume that instruction effective for students with LD may, with modifications, be effective for students with other types of cognitive disabilities. *Solve It!*, with modifications, was found to be effective for three adolescents with spina bifida in a single-subject research study (Mesler, 2004).

The modifications included using a slower pace of instruction, eliminating the estimation process from the problem solving routine, providing individual flip charts of the processes and strategies, and progressing from mastery of one-step problems to mastery of two-step problems. Three-step problems were eliminated. Students were instructed individually three times per week for about five weeks. Together, the instructor and student developed the visual representations for the problems. The math problem solving of all three students improved substantially following intervention.

Following instruction, these students also seemed to have more interest in school and seemed more motivated to learn. One student told the researcher/instructor (Mesler, 2004, p.70), "Now I know how to do math; no one ever showed me before. My teacher even called my mother and told her how much better I am doing. People always told me I couldn't do it. Now I see I can and I feel better about myself." Instructional programs like *Solve It!* can be adapted for students with different types of cognitive impairments that range from mild to severe.

Conclusion

Solve It! is a research-based program that makes mathematical problem solving easy to teach. Students are provided with the processes and strategies that make math problem solving easy to learn. With *Solve It!, s*tudents become successful and efficient problem solvers. They also gain a better attitude toward problem solving when they are successful and develop the confidence to persevere. Moving from textbook problems to real life math situations creates a challenge for students, and they begin to understand why they need to be good problem solvers. Research-based programs like *Solve It!* provide problem solving instruction that gives students the resources to solve authentic, complex mathematical problems they encounter in everyday life. Teachers who are knowledgeable about the research underlying effective instruction will be able to justify the instructional time spent on *Solve It!* in their classes. They will also be able to explain how the program complements and builds on the mathematics curriculum.

Curricular Materials

The curriculum, *Solve It!,* is available from Exceptional Innovations at http://www.exinn.net

Other curricular materials available for problem solving strategies are:
Woodward, J., & Stroh, M. (2005). Developing number sense. Longmont, CO: Sopris West.
Woodward, J., & Stroh, M. (2005). Making sense of rational numbers. Longmont, CO: Sopris West.
Woodward, J., & Stroh, M. (2005). Understanding algebraic expressions. Longmont, CO: Sopris West.

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