



## MATH PROBLEM SOLVING FOR UPPER ELEMENTARY STUDENTS WITH DISABILITIES

### ABOUT THE AUTHOR

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### MATH PROBLEM SOLVING FOR UPPER ELEMENTARY STUDENTS WITH DISABILITIES

The skills and strategies needed for successful mathematical problem solving start developing in the preschool years, when children acquire a basic conceptual understanding of the base 10 numerical system. During these early years, they typically develop the “number sense” needed to process and manipulate numerical information. In primary school, children continue to acquire mathematical concepts and are exposed to a variety of math problem types requiring addition and subtraction operations. Students in grades three, four, and five continue to apply and refine the skills and strategies necessary to solve real life mathematics problems. By middle school, students should be able to apply these skills and strategies effectively and efficiently in school, at home, and in the community.

However, many students, especially students with learning disabilities (LD), have difficulty solving math word problems because they often cannot decide what to do to solve the problem. Most textbooks are not very helpful when it comes to teaching students how to solve math problems. They typically provide a four-step formula: (a) read the problem, (b) decide what to do, (3) compute, and (4) check your answer. *Understanding* the problem is at the core of “reading” the problem. To understand the problem, students need to be able to represent the problem, which provides the basis for deciding what to do to solve the problem. From early on, most students acquire the skills and strategies needed to “read the problem” and “decide what to do” to solve it. Many students with LD or other cognitive impairments, however, do not easily acquire these skills and strategies. Therefore, they need explicit instruction in mathematical problem solving skills and strategies to solve problems in their math textbooks and in their daily lives.

This series of three briefs focuses on teaching primary, upper elementary, and middle school students with disabilities how to solve mathematical word problems. This brief deals specifically with math problem solving for upper elementary school students. At this developmental stage, students are expected to be able to solve problems like this one:

***The Art Club is having a cookie sale. Each box of cookies costs \$2.00. The first day Jennifer sold 6 boxes, Carlos sold 7, and Alex sold 3. How much did the Art Club make the first day of cookie sales?***

The following frequently asked questions provide the framework for each brief.

- What is mathematical problem solving?

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- How do good problem solvers solve math problems?
  - Why is it so difficult to teach students to be good math problem solvers?
  - What is the content of math problem solving instruction?
  - What are effective instructional procedures for teaching math problem solving?

Several validated practices are described, and a sample lesson is provided. Additionally, specific adaptations and accommodations are provided for students with other types of cognitive disabilities, such as spina bifida and Asperger's Syndrome.

### **What is mathematical problem solving?**

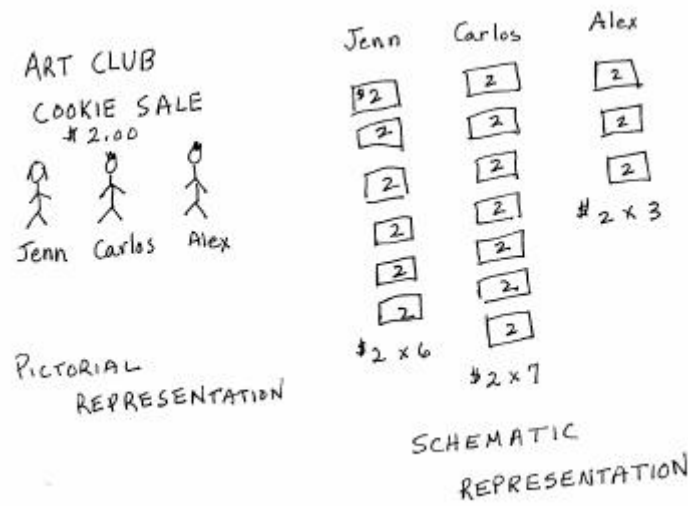
Mathematical problem solving is a complex cognitive activity involving a number of processes and strategies. Problem solving has two stages: problem representation and problem execution. Successful problem solving is not possible without first representing the problem appropriately. Appropriate problem representation indicates that the problem solver has understood the problem and serves to guide the student toward the solution plan. Students who have difficulty representing math problems will have difficulty solving them.

One of the most powerful problem representation strategies is visualization. Developmentally, for most children, visualization matures somewhere between the ages of 8 and 11. Therefore, students in upper elementary school should be able to use visualization effectively to represent mathematical problems. Students with LD, however, who have been characterized as having a variety of strategy deficits and differences, usually have difficulties using visualization as an effective learning strategy for remembering information and representing problems. Many students do not develop the ability to use visual representation automatically during math problem solving. These students need explicit instruction in how to use visualization to represent problems.

Teaching mathematical problem solving is a challenge for many teachers, many of whom rely almost exclusively on mathematics textbooks to guide instruction. Most mathematics textbooks simply instruct students to draw a picture or make a diagram using the information in the problem. Students with LD at the upper elementary level may be incapable of developing an appropriate representation of the problem for a variety of reasons. First, they are generally operating at a fairly concrete level. Second, they are poor at visual representation. As a result, symbolic representation may not be possible without explicit instruction that incorporates manipulatives and other materials that will help students move from a concrete to a more symbolic, schematic level. In other words, teachers must provide systematic, progressive, and scaffolded instruction that considers the students' cognitive strengths and weaknesses.

Students who have difficulty solving math word problems usually draw a picture of the problem without considering the relationships among the problem components and, as a result, still do not understand the problem and therefore cannot make a plan to solve it. So, it is not simply a matter of "drawing a picture or making a diagram;" rather, it is the type of picture or diagram that is important. Effective visual representations, whether with manipulatives, with paper and pencil, or in one's imagination, show the relationships among the problem parts. These are called schematic representations (van Garderen & Montague, 2003). Poor problem solvers tend to make immature representations that are more pictorial than schematic in nature. The illustration below shows the difference between a pictorial

and a schematic representation of the mathematical problem presented at the beginning of the brief.



Other cognitive processes and strategies needed for successful mathematical problem solving include paraphrasing the problem, which is a comprehension strategy, hypothesizing or setting a goal and making a plan to solve the problem, estimating or predicting the outcome, computing or doing the arithmetic, and checking to make sure the plan was appropriate and the answer is correct (Montague, 2003; Montague, Warger, & Morgan, 2000). Mathematical problem solving also requires self-regulation strategies. Students with LD are notoriously poor self-regulators. During this developmental period, it is imperative that they be explicitly taught how to self-instruct (tell themselves what to do), self-question (ask themselves questions), and self-monitor (check themselves as they solve the problem).

### What do good problem solvers do?

Good problem solvers use a variety of processes and strategies as they read and represent the problem before they make a plan to solve it (Montague, 2003). First, they **READ the problem for understanding**. As they read, they use comprehension strategies to translate the linguistic and numerical information in the problem into mathematical notations. For example, good problem solvers may read the problem more than once and may reread parts of the problem as they progress and think through the problem. They use self-regulation strategies by asking themselves if they understood the problem. They **PARAPHRASE the problem by putting it into their own words**. They identify the important information and may even underline parts of the problem. Good problem solvers ask themselves what the question is and what they are looking for. **VISUALIZING or drawing a picture or diagram** means developing a schematic representation of the problem so that the picture or image reflects the relationships among all the important problem parts. Using both verbal translation and visual representation, good problem solvers are not only guided toward understanding the problem, but are also guided toward developing a plan to solve the problem. This is the point at which students decide what to do to solve the problem. They have represented the problem and they are now ready to develop a solution path. They **HYPOTHESIZE** by thinking about logical solutions and the types of operations and number of steps needed to solve the problem. They may write the operation symbols as they decide on the most appropriate solution path and the algorithms

they need to carry out the plan. They ask themselves if the plan makes sense given the information they have. Good problem solvers usually **ESTIMATE or predict the answer** using mental calculations or may even quickly use paper and pencil as they round the numbers up and down to get a “ballpark” idea. They are now ready to **COMPUTE**. So they tell themselves to do the arithmetic and then compare their answer with their estimate. They also ask themselves if the answer makes sense and if they have used all the necessary symbols and labels such as dollar signs and decimals. Finally, they **CHECK** to make sure they used the correct procedures and that their answer is correct.

### **Why is it so difficult to teach students to solve math problems?**

Students who are poor mathematical problem solvers, as most students with LD are, do not process problem information effectively or efficiently. They lack or do not apply the resources needed to complete this complex cognitive activity. Generally, these students also lack metacognitive or self-regulation strategies that help successful students understand, analyze, solve, and evaluate problems. To help these students become good problem solvers, teachers must understand and teach the cognitive processes and metacognitive strategies that good problem solvers use. This is the **CONTENT** of math problem solving instruction. Teachers must also use instructional **PROCEDURES** that are research-based and that have been proven effective. These procedures are the basis of **COGNITIVE STRATEGY INSTRUCTION**, which has been demonstrated to be one of the most powerful interventions for students with LD (Swanson, 1999).

### **What is the content of math problem-solving instruction?**

The previous sections described the content of math problem-solving instruction as the cognitive processes and metacognitive strategies that good problem solvers use to solve mathematical problems. Students learn how to use these processes and strategies not only effectively, but efficiently as well. The following chart lists the processes and their accompanying self-regulation strategies that facilitate application of the processes (Montague, 2003). For upper elementary students, teaching needs to be systematic. At the outset, reading, paraphrasing, and visualizing should be emphasized. In the initial stages, manipulatives should be used to develop the representation. Based on this pictorial representation of three-dimensional objects, a schematic representation using paper and pencil is formed. Eventually the representation is transformed into a symbolic mathematical representation using mathematical notation.

#### **Math Problem-Solving Processes and Strategies**

**READ** (for understanding)

**Say:** Read the problem. If I don't understand, read it again.

**Ask:** Have I read and understood the problem?

**CHECK:** For understanding as I solve the problem.

**PARAPHRASE** (your own words)

**Say:** Underline the important information. Put the problem in my own words.

**Ask:** Have I underlined the important information? What is the question?  
What am I looking for?

**CHECK:** That the information goes with the question.

**VISUALIZE** (make a picture or a diagram and use manipulatives)

**Say:** Make a drawing or a diagram. Use manipulatives to show the relationships among the problem parts.

**Ask:** Does the picture fit the problem? Did I show the relationships?

**CHECK:** The picture against the problem information.

**HYPOTHESIZE** (a plan to solve the problem)

**Say:** Decide how many steps and operations are needed. Write the operation symbols (+, -, x, and /).

**Ask:** If I ..., what will I get? If I ..., then what do I need to do next? How many steps are needed?

**CHECK:** That the plan makes sense.

**ESTIMATE** (predict the answer)

**Say:** Round the numbers, do the problem in my head, and write the estimate.

**Ask:** Did I round up and down? Did I write the estimate?

**CHECK:** That I used the important information.

**COMPUTE** (do the arithmetic)

**Say:** Do the operations in the right order.

**Ask:** How does my answer compare with my estimate? Does my answer make sense? Are the decimals or money signs in the right places?

**CHECK:** That all the operations were done in the right order.

**CHECK** (make sure everything is right)

**Say:** Check the plan to make sure it is right. Check the computation.

**Ask:** Have I checked every step? Have I checked the computation? Is my answer right?

**CHECK:** That everything is right. If not, go back. Ask for help if I need it.

### What are effective instructional procedures for teaching math problem solving?

EXPLICIT INSTRUCTION, the basis of cognitive strategy instruction, incorporates research-based practices and instructional procedures such as cueing, modeling, verbal rehearsal, and feedback. The lessons are highly organized and structured. Appropriate cues and prompts are built in as students learn and practice the cognitive and metacognitive processes and strategies. Each student is provided with immediate, corrective, and positive feedback on performance. Overlearning, mastery, and automaticity are the goals of instruction. Explicit instruction allows students to be active participants as they learn and practice math problem solving processes and strategies. This approach emphasizes interaction among students and teachers.

Through an extensive and statistical review of the intervention studies conducted over 20 years with students with LD, Swanson (1999) identified the following eight components of effective strategy instruction. They are described as they would be used in teaching mathematical problem solving.

#### 1. SEQUENCING AND SEGMENTING

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Sequencing and segmenting means breaking the task into component subparts, providing short activities, and synthesizing the parts into a whole. For example, each cognitive process/self-regulation strategy routine is taught consecutively, beginning with reading the problem as a necessary first step for solving the problem. Students are taught to read the problem and then ask themselves if they understood it. They are then taught to go back and reread it or read parts until they decide they understand it. At the beginning of instruction, the teacher models the process and provides plenty of step-by-step cues and prompts as students practice. Eventually these cues and prompts are phased out. After students know what to do when they read math problems, they learn how to paraphrase problems. Students learn the paraphrasing routine, which is then added to the reading routine. At this point, students have mastered a sequence of two important processes for solving mathematical problems.

## 2. DRILL-REPETITION AND PRACTICE-REVIEW

This component includes daily tests to measure skill mastery, sequenced review, repeated practice, distributed review and practice, using the same or similar practice problems, and providing ongoing and positive feedback. For example, the paraphrasing routine is taught and then students practice on their own or with peers.

### **PARAPHRASE** (your own words)

**Say:** Underline the important information. Put the problem in my own words.

**Ask:** Have I underlined the important information? What is the question?  
What am I looking for?

**CHECK:** That the information goes with the question.

After the teacher models the routine and guides the students as they go through the routine, they are provided with practice until the routine becomes automatic. As they learn how to paraphrase math word problems, they can evaluate themselves using a checklist and plot their improvement on a graph.

## 3. DIRECTED QUESTIONING AND RESPONSES

Cognitive strategy instruction uses a guided discussion technique to promote active teaching and learning. Students are engaged from the very beginning through an initial discussion of the importance of mathematical problem solving. With the teacher, they set individual performance goals and make a commitment to becoming a better problem solver. Teachers ask both “process-related” and “content-related” questions. Students are directed by the teacher to ask questions. Students are also taught when and how to ask for help.

## 4. CONTROL DIFFICULTY OR PROCESSING DEMANDS OF THE TASK

Arrange tasks from easy to difficult. The teacher provides simplified demonstrations, necessary assistance, appropriate cues and prompts, and guided discussion. For upper elementary students, math problem solving instruction should start with one-step problems involving only whole numbers. When students have mastered the problem-solving routine with problems at this level, they can progress to one-step problems with decimals (e.g.,



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problems involving money). They can then progress to two-step problems with whole numbers, and so on. Some students may reach a threshold (e.g., two-step problems with decimals).

## 5. TECHNOLOGY

Technology extends beyond calculators and computers to include structured text, flow charts, structured curricula, scripted lessons, and video demonstrations. Students who are learning to be better math problem solvers should be taught how to use calculators to facilitate computation.

## 6. GROUP INSTRUCTION

Students with LD who have math problem-solving difficulties should be taught in small groups (5-8 students), maximizing interaction between teachers and students. Interaction between teachers and students and among peers is the cornerstone of cognitive strategy instruction. Cognitive strategy instruction is intensive and time-limited.

## 7. SUPPLEMENTS TO TEACHER AND PEER INVOLVMENT

Students are given cue cards to study for homework as they memorize and learn the various routines in the comprehensive strategy. These routines can be added to a cue card ring as they accumulate additional routines. That is, students begin with learning the “reading” routine. When they have mastered how to read a math word problem, they advance to the “paraphrasing” routine and add that cue card to their process/strategy ring. This is used as a homework exercise and during class as a support when students are solving problems independently. Students are expected to return to the general education math class and use what they have learned about solving math problems. General education teachers must be made aware of the instruction that students are receiving and must supplement and support this instruction in the general education math classes.

To do this, it is essential that general and special education teachers communicate regularly about the children and the instruction, and coordinate what is being taught in the general education class with what is taught in the special education resource class and vice versa. Continuity across general and special education is essential for student success. General education teachers must reinforce what students have learned to ensure that they apply this knowledge appropriately and also maintain acquired skills and strategies. See page 12 for an example of how a general education teacher and a special educator collaborate on instruction in math problem solving for their fourth grade students.

## 8. STRATEGY CUES

Students are given reminders and prompts such as individual Student Cue Cards to carry with them for home and class use, Master Class Charts on the classroom walls, think-aloud protocols, and discussion about the benefits of using strategies.

The next section presents several instructional procedures that are the basis of cognitive strategy instruction. These include verbal rehearsal, process modeling, visualization, role reversal, peer coaching, performance feedback, distributed practice, and mastery learning.

## VERBAL REHEARSAL

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Before students practice using the cognitive processes and self-regulation strategies, they must first memorize them by using verbal rehearsal. This is a memory strategy that enables students to automatically recall the math problem-solving processes and strategies. Students in upper elementary school can learn the **SAY, ASK, CHECK** routines as they are learning how to represent problems. Frequently, acronyms are created to help students remember as they verbally rehearse and internalize the labels and definitions for the processes and strategies. For math problem solving, the acronym **RPV-HECC** was created (**R** = Read for understanding, **P** = Paraphrase – in your own words, **V** = Visualize – draw a picture or diagram, **H** = Hypothesize – make a plan, **E** = Estimate – predict the answer, **C** = Compute – do the arithmetic, **C** = Check – make sure everything is right). Cues and prompts are used to help students as they memorize the processes and their definitions. The goal is for students to recite from memory all processes and name the self-regulation strategies (SAY, ASK, CHECK). When students have memorized the processes for math problem solving, they can cue other students and the teacher as they begin to use the processes and strategies to solve problems.

## PROCESS MODELING

Process modeling is thinking aloud while demonstrating an activity. For mathematical problem solving, this means that the problem solver says everything she or he is thinking and doing while solving a problem. When students are first learning how to apply the processes and strategies, the teacher demonstrates and models what good problem solvers do as they solve problems. Students have the opportunity to observe and hear how to solve mathematical problems. Both correct and incorrect problem-solving behaviors are modeled. Modeling of correct behaviors helps students understand how good problem solvers use the processes and strategies appropriately. Modeling of incorrect behaviors allows students to learn how to use self-regulation strategies to monitor their performance and locate and correct errors. Self-regulation strategies are learned and practiced in the actual context of problem solving. When students learn the modeling routine, they then can exchange places with the teacher and become models for their peers. Initially, students will need plenty of prompting and reinforcement as they become more comfortable with the problem-solving routine. However, they soon become proficient and independent in demonstrating how good problem solvers solve math problems. One of the instructional goals is to gradually move students from overt to covert verbalization. As students become more effective problem solvers, they will begin to verbalize covertly and then internally. In this way, they not only become more effective problem solvers, but they also become more efficient problem solvers.

## VISUALIZATION

Visualization is critical to problem representation. It allows students to construct an image of the problem on paper or mentally. Students must be shown how to select the important information in the problem and develop a schematic representation. To do this, teachers model how to use manipulatives to represent a problem, and then how to draw a picture or make a diagram that shows the relationships among the problem parts using both the linguistic and numerical information in the problem. These pictorial representations of three-dimensional objects and two-dimensional visual representations can take many forms, and will vary from student to student. Students may use a variety of visual representations such as manipulatives, pictures, tables, graphs, or other types of displays. Initially, students must be shown how to use the manipulatives and also how to translate the results of their



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manipulations with concrete objects to more symbolic representations using paper and pencil. Later, as students become more proficient, they will progress to mental images. Interestingly, if the problem is novel or challenging, they frequently return to conscious application of processes and strategies, which is typical of good problem solvers.

## ROLE REVERSAL

Role reversal is an important instructional activity that promotes independent learners. As students become familiar with the math problem-solving routine, they can take on the role of teacher as model and actually change places with the teacher. They may use an overhead projector just as the teacher did and engage in process modeling to demonstrate that they can effectively apply the cognitive and metacognitive processes and strategies they have learned. Other students can prompt or ask questions for clarification. In this way, students learn to think about, explain, and justify their visual representations and their solution paths. Teachers may also take the role of the student who then guides the “student as teacher” through the process. This interaction allows students to appreciate that there is usually more than one correct solution path for a math problem; that is, problems can be solved in a variety of ways.

## PEER COACHING

Peer partners, teams, and small problem-solving groups give students opportunities to see how their classmates approach mathematical problems differently, how they use cognitive and metacognitive processes and strategies differently, and how they represent and solve problems differently. Students gain a broader perspective on the problem-solving process and begin to realize that there is more than one way to solve a problem. Students become more flexible and tolerant thinkers as a result. With their partners or groups, students are encouraged to discuss the problems and work toward common solutions while appreciating the differences in approaches to each problem. This is also an opportunity to continue explaining and clarifying their choices. When students reach their performance goals and demonstrate mastery, novel or “real life” problems like the following can be introduced for the partners, teams, or small groups (Montague, 2003).

### **Novel Mathematical Problem for Partner, Team, or Group Problem Solving**

Your group is planning a picnic for your class. The class wants to have hot dogs, potato chips, watermelon, cookies, and soda. How much money is needed for the picnic?

## PERFORMANCE FEEDBACK

Performance feedback is critical to the success of the program. Progress checks are given throughout the program to determine mastery of the cognitive and metacognitive processes and strategies and performance on math problem solving tests. Students graph their progress to visually display their performance. Teachers carefully analyze performance during practice sessions and on mastery checks and provide each student with immediate, corrective feedback. Appropriate use of processes and strategies is reinforced continuously until students become proficient. Students need to know the specific behaviors for which they are praised so they can repeat these behaviors. Praise should be honest. Students should be taught how to give and receive reinforcement to others and themselves and

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should have plenty of opportunities to practice doing it. The goal is to teach students to monitor, evaluate, and reinforce themselves as problem solvers.

## DISTRIBUTED PRACTICE

Distributed practice is the cornerstone for ensuring that students maintain what they have learned. To become good math problem solvers, students learn to use the processes and strategies that successful problem solvers use. As a result, their math problem-solving skills and performance levels improve. However, to achieve high performance, students must be given ample opportunity to practice initially as they learn the math problem-solving routine and, then, to maintain high performance, they must continue to practice intermittently over time. They may practice individually or in teams or small groups. They should be involved in solving a range of problems from textbook type problems to problems encountered in real life. Discussion about strategies, error monitoring, and alternative solutions is essential.

## MASTERY LEARNING

Prior to instruction, a pretest is given to determine baseline performance levels of individual students. During instruction, periodic mastery checks are given to monitor student progress over time and to determine effectiveness of the program. If students are not making sufficient progress, modifications to the program to ensure success must be made. Following instruction, periodic maintenance checks are provided. If students do not meet criterion on maintenance checks, booster sessions must be provided to improve performance levels to mastery. Booster sessions are brief lessons to review and refresh what students have previously learned and mastered.

### **Teaching Math Problem Solving to Upper Elementary Students with LD**

Consider Ms. Hutson's fourth grade math class. She has 28 students in her class. Six have identified learning disabilities and receive resource room support. These students have considerable difficulty solving math word problems. Ms. Hutson notices that another eight students are also having difficulty solving math word problems. She decides to seek help from the special education resource teacher, Ms. Elias. They decide to work together on improving students' math problem-solving skills. The resource teacher will teach the six students with LD during their resource time. Ms. Hutson will provide small group instruction for the eight students from her class during independent seatwork time. This coincides with the resource period for the students with LD. They develop structured lessons and create the necessary cues and prompts. They make a plan to meet every other day for about 20 minutes to identify what's working and what needs improvement in their lessons, and also to appraise the progress of individual students.

Ms. Hutson and Ms. Elias have progressed through the Reading and Paraphrasing routines. They are now ready to start the Visualization routine. Their meetings have been very helpful. They have talked through a number of concerns and brainstormed about techniques that will help some of the students who are having difficulties. All of the students are making progress, although some need additional assistance, which is provided primarily by peers during peer coaching time at the end of the day. Ms. Hutson will oversee and monitor the peer coaching. Below is a structured lesson designed to teach students how to develop schematic representations of math problems. This is the first day of instruction in visual representation. She places a transparency of the math problem on the projector.

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**Ms. Hutson:** Watch me say everything I am thinking and doing as I solve this problem.

There are 72 cookies on the tray. Maria takes 28 cookies for her class. How many cookies are left on the tray?

First, I am going to **read** the problem for understanding.

SAY: Read the problem. Okay, I will do that. (Ms. Hutson reads the problem.) If I don't understand it, I will read it again. Hm, what is happening? I think I need to read it again. (She reads the problem again.) Oh, I get it. There are 72 and she takes 28.

ASK: Have I read and understood the problem? I think so.

CHECK: For understanding as I solve the problem. Okay, I understand it.

Next, I am going to **paraphrase** by putting the problem into my own words.

SAY: Put the problem into my own words. The girl takes 28 cookies from the tray. The tray has 72. How many are left? Underline the important information. I will underline 72 cookies and 28 cookies.

ASK: Have I underlined the important information? Let's see, yes I did. What is the question? The question is "how many cookies are left on the tray?" What am I looking for? I am looking for the number of cookies left.

CHECK: That the information goes with the question. I have the number of cookies on the tray and the number of cookies that the girl takes for her class. I need to find out the number left on the tray.

Then I will **visually represent the problem**.

SAY: Make a drawing or a diagram and use my base-10 blocks. Here goes. (She draws a big tray and puts 7 sets of 10-blocks and two single blocks in the tray. She draws a girl marching toward the tray with a bubble that says, "I need 28 of those cookies.")

ASK: Does the picture fit the problem? Yes, I believe it does tell the story.

CHECK: The picture against the problem information. Let me make sure I have the correct numbers. (She counts the blocks. Yes, 72. She writes 72. And 28. Yes, she needs 28 of the cookies for her class).

Students then review the Master Class Charts and are given the routine to add to their cue card ring. Then they are given a problem and told to read, paraphrase, and visualize the problem just as the teacher did. They must think out loud just as the teacher did. A student is then selected to model the process with assistance from Ms. Hutson. Notice that students learn the problem-solving processes and strategies at only the representation phase of mathematical problem solving. They are not yet at the problem solution phase. Instruction is systematic, sequenced, slow but intense, and structured to ensure mastery as students learn the processes and strategies. Although Ms. Hutson and Ms. Elias were successful in their collaborative effort to help students become better problem solvers and they know the program was effective for their students, they have several caveats for other teachers.

### **Caveats for Teaching Math Problem Solving**

- Individualizing instruction may be difficult given the large numbers of students enrolled in most elementary school classes. Class size can range from 20 to 35 students. Enlisting the aid of the resource teacher to assist with individualized and small group instruction may be necessary but not always feasible. *One solution may be to hire paraprofessionals or enlist the aid of volunteers to assist classroom teachers. Teachers often have these individuals work with low-performing students. It is preferable to have them monitor and assist higher performing students, which frees the classroom teacher*

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*to work directly with students who are having difficulty. In this way, the teacher can re-teach concepts, skills, and strategies and reinforce what has been taught.*

- Identifying the students who need instruction and then grouping for instruction based on the various levels in the class can be a challenge for a math teacher. *There are many easily administered formal and informal measures that provide relatively valid performance levels. Additionally, basal series usually include curriculum-based measures that provide measures of baseline performance and progress over time.*
- General education math teachers often feel unprepared to teach students who are in special programs. They may not feel confident that students can learn how to think differently and become good problem solvers. *If teachers feel the need for more knowledge and skills in teaching mathematics, particularly problem solving, to students with special needs, they should submit a request in writing to their administrator(s) for professional development training in mathematics. Special educators should attend this training with general educators, as this could be an opportunity to develop a long-term plan for collaboration between general and special education.*
- Finding time to talk with the resource teacher for students in special education can be difficult. Also, teachers often do not coordinate resource room instruction with the general education math curriculum. Communication between teachers can sometimes be difficult. *Administrators need to be involved in order to set aside time for teachers to discuss students and the coordination of instruction. Special educators should be involved in curriculum meetings, grade level meetings, and, of course, child study team meetings to facilitate communication and coordination.*
- Teachers may need to develop the knowledge and skills to implement mathematical problem solving instruction successfully. Because the program is intense and highly interactive, teachers may need professional development to learn the instructional procedures that are the foundation of cognitive strategy instruction. *Again, administrators must be informed about the need for professional development.*
- Teachers may not be familiar with the research that supports cognitive strategy instruction, nor with the strategy's instructional procedures. *Professional development should give teachers both a background in research-based practice and also the knowledge about its implementation.*

### **Modifying Math Problem-Solving Instruction for Students with Other Types of Disabilities**

Students with other types of disabilities frequently display cognitive characteristics that resemble those of students with LD. However, their cognitive deficits may be more or less severe or may vary in some unique way from those of students with LD. In many cases, though, there seem to be more similarities than differences. For example, students with spina bifida have long-term memory, visual-spatial, and self-regulation problems that adversely affect their ability to comprehend text and do mathematics (Mesler, 2004). Children with chronic illnesses who have undergone intrusive medical treatments (e.g., children surviving cancer) often display attention difficulties, short-term memory loss, and other cognitive problems that interfere with school success (Bessell, 2001). Students with traumatic brain injury and Asperger's Syndrome also have cognitive deficits that are similar to students with LD. Because of these similarities, it seems reasonable to assume that instruction effective for students with LD may, with modifications, be effective for students with other types of cognitive disabilities.

A problem-solving curriculum, *Solve It!*, was modified for three adolescents with spina bifida and found to be effective (Mesler, 2004). In this program designed for adolescents, the

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modifications included using a slower pace of instruction, eliminating the estimation routine from the comprehensive problem-solving routine, providing individual flip charts of the processes and strategies, and progressing from mastery of one-step problems to mastery of two-step problems. Three-step problems were eliminated. Students were instructed individually three times per week for about five weeks. Together, the instructor and student developed the visual representations for the problems. The math problem solving of all three students improved substantially following the intervention.

Following instruction, these students also seemed to have more interest in school and seemed more motivated to learn. One student told the researcher/instructor (Mesler, 2004, p.70), "Now I know how to do math; no one ever showed me before. My teacher even called my mother and told her how much better I am doing. People always told me I couldn't do it. Now I see I can and I feel better about myself." Cognitive strategy instructional programs can be adapted for students of all ages with different types of cognitive impairments that range from mild to severe.

### **Conclusion**

A systematic, research-based math problem-solving program makes mathematical problem solving easy to teach. Students are provided with the processes and strategies that make math problem solving easy to learn, and they become successful and efficient problem solvers. They also gain a better attitude toward problem solving when they are successful, and they develop the confidence to persevere. Moving from textbook problems to real life math situations creates a challenge for students, and they begin to understand why they need to be good problem solvers. Cognitive strategy instruction in mathematical problem solving gives students the resources to solve authentic, complex mathematical problems they encounter in everyday life. Teachers who are knowledgeable about the research underlying effective instruction will be able to justify the instructional time spent on small group instruction in math problem solving. They will also be able to explain how the supplemental instruction complements and builds on the mathematics curriculum.

## References

Bessell, A. (2001). Children surviving cancer: Psychosocial adjustment, quality of life, and school experiences. *Exceptional Children*, 67, 345-359.

Mesler, J.L. (2004). *The effects of cognitive strategy instruction on the mathematical problem solving of students with spina bifida*. Unpublished doctoral dissertation, University of Miami, Coral Gables, Florida.

Montague, M. (2003). *Solve It! A practical approach to teaching mathematical problem-solving skills*. Reston, VA: Exceptional Innovations.

Montague, M., Warger, C.L., & Morgan, H. (2000). Solve It! Strategy instruction to improve mathematical problem solving. *Learning Disabilities Research and Practice*, 15, 110-116.

Swanson, H.L. (1999). Instructional components that predict treatment outcomes for students with learning disabilities: Support for a combined strategy and direct instruction model. *Learning Disabilities & Research*, 14, 129-140.

van Garderen, D., & Montague, M. (2003). Visual-spatial representations and mathematical problem solving. *Learning Disabilities & Research*, 18, 246-254.

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